Comprehensive Test

2009

Sponsored by the Indiana Council of Teachers of Mathematics

Indiana State Mathematics Contest

This test was prepared by **Indiana University-Purdue University Indianapolis**

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ICTM Website
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Next year's math contest date: April 24, 2010

1. If $y = \log_a x$, and $a > 1$, which of the following statements is incorrect?

(A) if $x = 1$, then $y = 0$
(B) if $x = a$, then $y = 1$
(C) if $x = -1$, then y is imaginary
(D) if $0 < x < 1$, then y is always less than 0 and decreases without limit as x approaches zero
(E) only some of the above statements are correct

2. How many faces, edges, and vertices does a pyramid with an n-gon base have?

(A) n, n+1, n+2
(B) n+1, 2n, n+1
(C) n, 2n, n+1
(D) n+1, n, 2n
(E) 2n, n+1, n+2

3. The sum of the roots of the equation $4x^2 - 8x + 5 = 0$ is equal to:

(A) 8
(B) -5
(C) $\frac{-5}{4}$
(D) -2
(E) 2

4. The limit of $\frac{x^2 - 1}{x - 1}$ as x approaches 1 as a limit is:

(A) 0
(B) indeterminate
(C) $x - 1$
(D) 2
(E) 1

5. The fraction $\dfrac{\sqrt{a^2+x^2}-\left(x^2-a^2\right)/\sqrt{a^2+x^2}}{a^2+x^2}$ reduces to:

(A) 0
(B) $\dfrac{2a^2}{a^2+x^2}$
(C) $\dfrac{2x^2}{\left(a^2+x^2\right)^{3/2}}$
(D) $\dfrac{2a^2}{\left(a^2+x^2\right)^{3/2}}$
(E) $\dfrac{2x^2}{a^2+x^2}$

6. The Fibonacci sequence is defined by $F_1=1,\ F_2=1,\ F_n=F_{n-1}+F_{n-2}$ for $n\geq 2$. Determine the smallest positive integer k so that F_k is divisible by 31.

(A) 8
(B) 15
(C) 30
(D) 60
(E) none of these

7. How many pairs (m,n) of integers satisfy the equation $m+n=mn$?

(A) 1
(B) 2
(C) 3
(D) 4
(E) more than 4

8. Simplify $\left(i\sqrt{2}-\sqrt{2}\right)^{10}$.

(A) 1024
(B) $1024i$
(C) -1024
(D) $-1024i$
(E) 32

9. If $\begin{vmatrix} a & c \\ d & b \end{vmatrix}$ has the value $ab - cd$, then the equation $\begin{vmatrix} 2x & 1 \\ x & x \end{vmatrix} = 3$:

(A) is satisfied for only 1 value of x.
(B) is satisfied for 2 values of x.
(C) is satisfied for no values of x.
(D) is satisfied for an infinite number of values of x.
(E) none of these.

10. In our number system the base is ten. If the base were changed to five you would count as follows: 1, 2, 3, 4, 10, 11, 12, 13, 14, 20, 21, ... The thirtieth number would be:

(A) 30
(B) 40
(C) 50
(D) 55
(E) 110

11. Determine m so that $4x^2 - 6x + m$ is divisible by $x - 3$. The obtained value, m, is an exact divisor of:

(A) 12
(B) 20
(C) 36
(D) 48
(E) 64

12. If $f(a) = a - 2$ and $F(a,b) = b^2 + a$, then $F\big(3, f(4)\big)$ is:

(A) $a^2 - 4a + 7$
(B) 28
(C) 7
(D) 8
(E) 11

13. There are two cards; one is red on both sides and the other is red on one side and blue on the other. The cards have the same probability of being chosen, and one is chosen and placed on the table. If the upper side of the card on the table is red, then the probability that the under-side is also red is

(A) 1 / 4
(B) 1 / 3
(C) 1 / 2
(D) 2 / 3
(E) 3 / 4

14. A four-member senate committee is to be formed from the one hundred United States senators (two from each of the fifty states). How many different committees can be formed if all four members must be from different states?

(A) 4
(B) 230,300
(C) 3,684,800
(D) 5,527,200
(E) None of these

15. Evaluate $\tan\left(\arccos\left(\frac{-3}{4}\right)\right)$.

(A) $\frac{-\sqrt{7}}{3}$
(B) $\frac{-3}{\sqrt{7}}$
(C) $\frac{3}{\sqrt{7}}$
(D) $\frac{4}{3}$
(E) $\frac{\sqrt{7}}{3}$

16. The equation $\sqrt{x+4}-\sqrt{x-3}+1=0$ has:

(A) no root
(B) one real root
(C) one real root and one imaginary root
(D) two imaginary roots
(E) two real roots

17. Simplify $\dfrac{\cot\theta - \tan\theta}{\cot\theta + \tan\theta}$.

(A) $\cos 2\theta$
(B) $-\cos 2\theta$
(C) $1 - 2\cos^2\theta$
(D) $\sin 2\theta$
(E) none of these

18. In the following equation, each of the letters represents uniquely a different digit in base ten: $(YE)\cdot(ME) = TTT$. The sum $E + M + T + Y$ equals

(A) 19
(B) 20
(C) 21
(D) 22
(E) 24

19. The sum of all numbers of the form $2k+1$, where k takes on integral values from 1 to n is:

(A) n^2
(B) $n(n+1)$
(C) $n(n+2)$
(D) $(n+1)^2$
(E) $(n+1)(n+2)$

20. The area of a circle inscribed in a regular hexagon is 100π. The area of the hexagon is:

(A) 600
(B) 300
(C) $200\sqrt{2}$
(D) $200\sqrt{3}$
(E) $120\sqrt{5}$

21. Determine the largest positive integer n such that 1005! is divisible by 10^n.

(A) 102
(B) 112
(C) 249
(D) 250
(E) 502

22. The roots of $Ax^2 + Bx + C = 0$ are r and s. For the roots of $x^2 + px + q = 0$ to be r^2 and s^2, p must equal:

(A) $\dfrac{B^2 - 4AC}{A^2}$
(B) $\dfrac{B^2 - 2AC}{A^2}$
(C) $\dfrac{2AC - B^2}{A^2}$
(D) $B^2 - 2C$
(E) $2C - B^2$

23. If $f(x) = \sqrt{x-2}$ and $g(x) = \dfrac{1}{x+1}$, determine the domain of $(f \circ g)(x)$.

(A) $-1 < x \le \dfrac{-1}{2}$
(B) $-1 \le x \le \dfrac{-1}{2}$
(C) $\dfrac{-1}{2} \le x \le 2$
(D) $-1 \le x$
(E) $2 \le x$

24. For which value of n will the matrix $\begin{bmatrix} 2 & 9 & -1 \\ 0 & 4 & n \\ 0 & 1 & 3 \end{bmatrix}$ have a determinant of zero?

(A) 0
(B) 1
(C) 3
(D) 9
(E) 12

25. How many circular pipes with inside diameter of 1 inch will carry the same amount of water as a pipe with an inside diameter of 6 inches?

(A) 3
(B) 6
(C) 12
(D) 36
(E) 48

26. If $\log_k x \cdot \log_5 k = 3$, then x equals:

(A) k^5
(B) $5k^3$
(C) k^3
(D) 243
(E) 125

27. A right triangle ABC has its hypotenuse AB trisected at M and N. If $CM^2 + CN^2 = k \cdot AB^2$, then what is the value of k?

(A) $\frac{5}{9}$
(B) $\frac{2}{3}$
(C) $\frac{1}{2}$
(D) 2
(E) $\frac{1}{4}$

28. The equation $x^{x^{x^{\cdots}}} = 2$ is satisfied when x is equal to:

(A) infinity
(B) 2
(C) $\sqrt[4]{2}$
(D) $\sqrt{2}$
(E) 1