

# Geometry/Integrated Math II

## 2009

Sponsored by the Indiana Council of Teachers of Mathematics

Indiana State Mathematics Contest

This test was prepared by faculty at **Indiana State University**

**ICTM Website**

**<http://www.indiana math.org/>**

Do not open this test booklet until you have been advised by the test proctor.

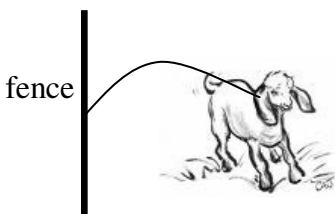
Next year's math contest date: April 24, 2010

- 1) Which of the following sets of 3 segments would make up the three sides of an isosceles triangle?
- I. 3 cm – 3 cm – 3 cm
  - II. 3 cm – 2 cm – 3 cm
  - III. 2 cm – 5 cm – 4 cm
  - IV. 2 cm – 5 cm – 2 cm
- a. II and IV only
  - b. I, II, and IV only
  - c. I, II and III only
  - d. III only
  - e. none of these
- 2) What is the area, in square units, of the triangle whose vertices have coordinates (0, 6), (0, 2), and (4, -2)?
- a. 8
  - b. 16
  - c. 4
  - d. 32
  - e. none of these
- 3) The ratio of the length of the side of a square to the length of its diagonal is
- a.  $\sqrt{2} : 1$
  - b.  $\sqrt{2} : 2$
  - c.  $1 : \sqrt{3}$
  - d.  $\sqrt{3} : 1$
  - e. none of these
- 4) Which of the following sets of 3 segments would make up the three sides of an obtuse triangle?
- I. 3 cm – 4 cm – 5 cm
  - II. 4 cm – 4 cm – 6 cm
  - III. 3 cm – 4 cm – 6 cm
  - IV. 3 cm – 3 cm – 5 cm
- a. III only
  - b. I and IV only
  - c. III and IV only
  - d. II, III, and IV only
  - e. none of these
- 5) The product of the altitudes of the triangle whose vertices have coordinates (0,0), (6,0), and (3,3) is
- a.  $9\sqrt{2}$
  - b. 27
  - c.  $27\sqrt{2}$
  - d. 54
  - e. none of these

- 6) The sum of the number of vertices, edges, and faces of a regular octahedron is
- a. 14      b. 20      c. 23      d. 26      e. none of these
- 7) The ratio of the volume of a cube with side lengths equal to one unit to the volume of a square pyramid whose base is one square unit and whose height is one unit is
- a. 1 : 3      b. 3 : 1      c. 1 : 2      d. 2 : 1      e. none of these
- 8) A right circular cone is made from a circular sector of radius 5 cm and central angle equal to  $216^\circ$ . The volume of the cone, in cubic centimeters is
- a.  $\frac{5}{3}\pi$       b.  $2\pi$       c.  $\frac{25}{3}\pi$       d.  $12\pi$       e. none of these
- 9) Which of the following statements are true?
- I.      Some rectangles are squares.  
II.     All squares are rhombi.  
III.    Some kites are parallelograms
- a. I and II only  
b. I and III only  
c. II and III only  
d. I, II, and III  
e. none of these
- 10) The equation of a circle of radius 5 and center  $(-1, 3)$  is
- a.  $(x+1)^2 + (y-3)^2 = 25$   
b.  $(x-1)^2 + (y+3)^2 = 25$   
c.  $(x+1)^2 + (y-3)^2 = 5$   
d.  $(x-1)^2 + (y+3)^2 = 5$   
e. none of these

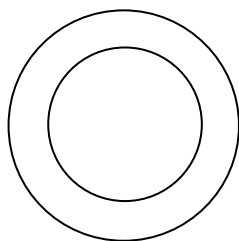
- 11) A goat is tethered to the middle of a 12 foot fence with a 10 foot length of chain, as shown in the diagram (not necessarily to scale). The area, in square feet, over which the goat is able to graze is

- a.  $50\pi$       b.  $66\pi$       c.  $100\pi$       d.  $116\pi$       e. none of these



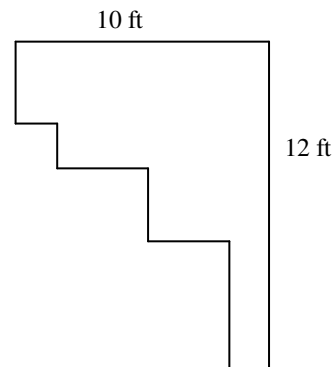
- 12) You are making a dart board by drawing two concentric circles, as shown in the diagram. What should the diameter of the large circle be, in centimeters, if the area of the ring is to equal the area of the small circle? The diameter of the small circle is 20 centimeters.

- a.  $10\sqrt{2}$   
 b.  $20\sqrt{2}$   
 c. 40  
 d.  $40\sqrt{2}$   
 e. none of these



- 13) Find the perimeter of the adjacent decagon. All angles are right angles.

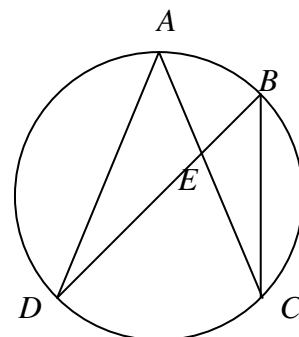
- a. 22 feet  
 b. 44 feet  
 c. 66 feet  
 d. 88 feet  
 e. none of these



- 14) The drawing below is of a circle with inscribed angles  $\angle A$ ,  $\angle B$ ,  $\angle C$ , and  $\angle D$ . Which of the following statements are true?

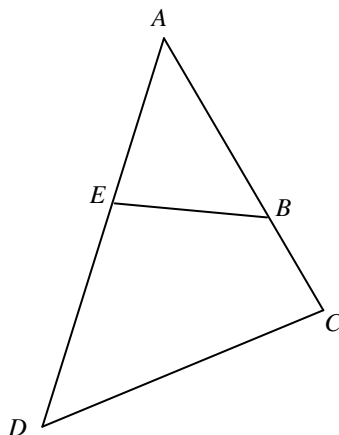
- I.  $\triangle AED \cong \triangle BEC$       II.  $\angle CAD \cong \angle CBD$   
 III.  $\triangle EAD \sim \triangle ECB$       IV.  $\frac{AB}{CD} = \frac{DE}{BE}$

- a. I only  
 b. II and IV only  
 c. I and II only  
 d. II, III, and IV only  
 e. none of these



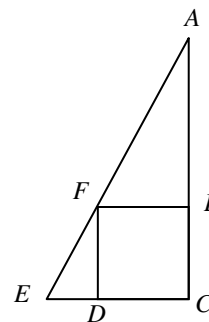
- 15) In the following diagram (not necessarily to scale),  $\angle ABE \cong \angle ADC$ ,  $AE = 6$ ,  $BC = 2$ ,  $BE = 3$ , and  $CD = 5$ .  $AB + DE$  is equal to

- a.  $\frac{46}{3}$   
 b.  $\frac{112}{3}$   
 c.  $\frac{13}{2}$   
 d. 20  
 e. none of these



- 16) In the adjacent diagram (not necessarily to scale), quadrilateral  $BCDF$  is a square,  $\angle AEC = 60^\circ$ , and  $AF = 6$ . The area of  $\triangle ACE$  is

- a.  $9\sqrt{3}$   
 b.  $3 + 3\sqrt{3}$   
 c.  $9 + 3\sqrt{3}$   
 d.  $9 + 6\sqrt{3}$   
 e. none of these



- 17) In a rhombus, one diagonal is twice the length of the other diagonal. Express the length of a side of the rhombus in terms of  $A$ , where  $A$  is the area of the rhombus.

- a.  $\frac{\sqrt{5A}}{2}$     b.  $\frac{\sqrt{4A}}{2}$     c.  $\frac{\sqrt{3A}}{2}$     d.  $\frac{\sqrt{2A}}{2}$     e. none of these

- 18) If the area of a rectangle is 100 cm, the smallest value of a diagonal, in centimeters, is

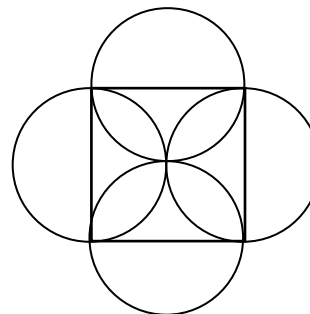
- a. 0    b. 10    c.  $\sqrt{200}$     d. 20    e. none of these

- 19) If an arc of  $72^\circ$  on circle  $A$  has the same length as an arc of  $120^\circ$  on circle  $B$ , then the ratio the area of circle  $A$  to the area of circle  $B$  is

- a. 25 : 9    b. 9 : 25    c. 5 : 3    d. 3 : 5    e. none of these

- 20) A figure is constructed by placing four congruent circles with a diameter equal to 4 meters, centered at the midpoints of the sides of a square of area 16 square meters. The area of the regions in which the four circles overlap, in square meters, is

- a.  $2\pi - 4$   
 b.  $4\pi - 4$   
 c.  $8\pi - 16$   
 d.  $16\pi - 16$   
 e. none of these

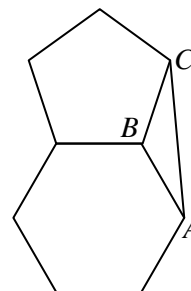


- 21) The sum of the measures of the exterior angles of a convex pentagon is

- a.  $180^\circ$       b.  $360^\circ$       c.  $540^\circ$       d.  $720^\circ$       e. none of these

- 22) The figure to the right is a regular hexagon with side length equal to  $s$  adjacent to a regular pentagon with side length also equal to  $s$ . The measure of  $\angle BAC$  is

- a.  $24^\circ$   
 b.  $48^\circ$   
 c.  $108^\circ$   
 d.  $132^\circ$   
 e. none of these



- 23) Triangle  $ABC$  is inscribed in a circle. The measure of the non-overlapping minor arcs  $AB$ ,  $BC$ , and  $CA$  are, respectively  $x + 105^\circ$ ,  $2x + 10^\circ$ , and  $3x - 25^\circ$ . The measure of one interior angle of the triangle is

- a.  $45^\circ$       b.  $50^\circ$       c.  $60^\circ$       d.  $100^\circ$       e. none of these

- 24) In quadrilateral  $ABCD$  with diagonals  $AC$  and  $BD$  intersecting at  $O$ ,  $BO = 3$ ,  $OD = 10$ ,  $AO = 5$ ,  $OC = 6$ , and  $AB = 6$ .  $BC$  is equal to

- a.  $\sqrt{\frac{71}{2}}$       b.  $\sqrt{\frac{143}{2}}$       c. 6      d.  $\sqrt{143}$       e. none of these

- 25) Greta is constructing a natural gas tank by attaching two hemispheres to the bases of a right circular cylinder. The diameter of the cylinder and sphere is 2 meters and the height of the cylinder is 2 meters. If she wants to paint the exterior of the tank and the paint she intends to use will cover 10 square meters per can, how many cans of paint should she buy to complete the job?
- a. 2            b. 3            c. 4            d. 5            e. none of these
- 26)  $(1, 1)$ ,  $(-3, -1)$  and  $(3, -2)$  are coordinates of three vertices of a parallelogram. The product of the coordinates of the fourth vertex, located in the third quadrant, is
- a. -10            b. 0            c. 4            d. 10            e. none of these
- 27) The surface area of a frustrum (a truncated right circular cone) with bases of radius 5 centimeters and 10 centimeters and a height of 12 centimeters is, in square centimeters,
- a.  $125\pi$             b.  $175\pi$             c.  $320\pi$             d.  $420\pi$             e. none of these
- 28) The distance between the two parallel lines which have equations  $y = 2x - 3$  and  $y = 2x + 2$  is
- a.  $\sqrt{5}$             b.  $\frac{5}{2}$             c. 5            d. 10            e. none of these
- 29) The hypotenuse of a right triangle has length  $\sqrt{41}$ . The sum of the lengths of the legs is 9. The area of the triangle is
- a. 10            b. 9            c. 8.5            d. 8            e. none of these
- 30) If the diagonals of a quadrilateral are perpendicular to each other, the figure would always be included under the general classification:
- a. rhombus    b. kite            c. square            d. trapezoid    e. none of these

- 31) The number of diagonals that can be drawn in a convex polygon of 50 sides is
- a. 1175      b. 1200      c. 1225      d. 1250      e. none of these
- 32) You have to replace a large pipe with inner diameter of 12 inches with smaller pipes of inner diameter 3 inches. How many of the 3 inch-diameter pipes would you need if you want to maintain the same flow through your system?
- a. 4      b. 6      c. 8      d. 16      e. none of these