

Comprehensive Test

2006

Sponsored by the Indiana Council of Teachers of Mathematics

Indiana State Mathematics Contest

This test was prepared by faculty at Indiana University Kokomo

ICTM Website

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Next year's math contest date: April 21, 2007

1. Let $f(x) = \frac{x}{x+1}$, and $g(x) = \frac{x+1}{x}$. Compute $(f \circ g)(x)$.

a. 1 b. 2 c. x

d. $\frac{2x+1}{x}$ e. $\frac{x+1}{2x+1}$

2. Find all the horizontal and vertical asymptotes of $f(x) = \frac{\sqrt{x^2 - 4}}{x + 2}$.

a. $x = -2, y = 0$ b. $x = -2, y = 1$ c. $x = -2, y = \pm 1$

d. $x = \pm 1, y = -2$ e. $x = 2, y = 1$

3. Find x in radians accurately to three decimal places such that

$6(\cos^2 x + 1) = -13 \cos x$ and $\pi \leq x \leq 2\pi$.

a. $x = 0.841, 2.301$ b. $x = 3.983, 5.442$ c. $x = 2.301$

d. $x = 3.983$ e. $x = 5.442$

4. A red die is tossed on one table and two blue dice are tossed on another table. What is the probability that the red die and the sum of the two blue dice are both 5 or both 6?

a. $\frac{5}{216}$ b. $\frac{1}{33}$ c. $\frac{1}{24}$

d. $\frac{5}{108}$ e. $\frac{7}{12}$

5. Which of the following are solutions of $x^4 = -1$?

a. $\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}$ b. $\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4}$ c. $\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}$

d. None of the above e. All of the above

6. Four sisters belong to a twelve person volleyball team. The team forms a line for a team picture. If the four sisters demand to stand together, how many different ways can the team line up for the picture?
- a. 362,880 b. 967,880 c. 1,451,520
d. 8,709,120 e. 479,001,600
7. Compute $\frac{90! - 88!}{88!}$.
- a. $\frac{1}{44}$ b. 178 c. 8009
d. 8010 e. $90! - 1$
8. If $xy = 5$ and $x^2y + xy^2 - x - y = 48$, find $x^2 + y^2$.
- a. 82.16 b. 92.16 c. 134
d. 144 e. 154
9. Solve for x : $2|x - 5| = |x + 5|$.
- a. $x = -5, -15$ b. $x = \frac{-5}{3}, -15$ c. $x = \frac{5}{3}, 15$
d. $x = 5, 15$ e. $x = 15$
10. Find all possible values of p such that $(-1, 5)$, $(0, p)$, and $(p, 8)$ are on a line.
- a. $p = -2, 4$ b. $p = -2, 6$ c. $p = 2 \pm 2\sqrt{3}$
d. $p = 3 \pm \sqrt{17}$ e. $p = 5$

11. Find the center of the graph of $-4x^2 + 9y^2 + 36y + 8x + 68 = 0$.

- a. $(1, -2)$ b. $(-1, 2)$ c. $(-2, 1)$
d. $(2, -1)$ e. $(1, 2)$

12. Find the product $(-1 + i\sqrt{3})(2\sqrt{3} + 2i)$.

- a. $4\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$ b. $8\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$
c. $4\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$ d. $8\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$
e. $8\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)$

13. Let $f(x) = \sqrt{x^2 + 1}$ for $|x| \geq 1$, and $g(x) = \sqrt{4 - x^2}$ for $|x| \leq 2$.
Find the domain of $(f + g)(x)$.

- a. $[-2, -1] \cup [1, 2]$ b. $[-2, 2]$ c. $(-2, 2)$
d. $(-2, -1) \cup (1, 2)$ e. None of the above

14. Solve for x : $3^{2x} - 3^x - 20 = 0$.

- a. $x = -2, \frac{5}{2}$ b. $x = -4, 5$ c. $x = \frac{\ln 3}{\ln 5}$
d. $x = \frac{\ln 5}{\ln 3}$ e. NO SOLUTION

15. If $\csc x = -5$ and $\tan x < 0$, then $\cos x =$

- a. $\frac{24}{25}$ b. $\frac{-1}{5}$ c. $\frac{2\sqrt{6}}{5}$
d. $\frac{\sqrt{26}}{5}$ e. $\frac{-2\sqrt{6}}{5}$

16. The numbers 6, x , y , $\frac{-16}{9}$ form a geometric progression. Find $x - y$.

- a. $\frac{-20}{3}$ b. $\frac{20}{3}$ c. $\frac{4}{3}$
d. $\frac{-4}{3}$ e. $\frac{8}{9}$

17. The domain of the function $f(x) = \log_5(x^2 - 9x + 14)$ is

- a. All real numbers b. $\{x \mid x \neq 2, 7\}$ c. $\{x \mid 2 < x < 7\}$
d. $\{x \mid x > 0\}$ e. $\{x \mid x < 2 \text{ or } x > 7\}$

18. Simplify $i^{2006} - i^{2005}$, where $i = \sqrt{-1}$.

- a. $1+i$ b. $1-i$ c. $-1+i$
d. $-1-i$ e. 0

19. If Λ represents an operation defined by $a \Lambda b = b^a + ab$, find $(2 \Lambda 3) \Lambda 1$.

- a. 14 b. 15 c. 16
d. 28 e. 30

20. A survey of 300 high school students regarding their music preferences determined that 140 liked rap music, 115 liked country music, and 125 liked rock music, while only 50 liked all three. It was also determined that 40 liked only rap music, 25 liked only country music, and 45 liked only rock music. How many liked none of these types of music?
- a. 60 b. 80 c. 140
d. 110 e. There is insufficient information given to answer the question.
21. Simplify $\frac{\cot x - \tan x}{\cot x + \tan x}$
- a. $\cos 2x$ b. $-\cos 2x$ c. $1 - 2\cos^2 x$
d. $\sin 2x$ e. None of the above
22. The graph of $y = f(x)$ is translated to the left 2 units. The equation of the resulting graph is
- a. $y = f(x-2)$ b. $y = f(x+2)$ c. $y = f(2x)$
d. $y = f(x) - 2$ e. $y = f(x) + 2$
23. If $f(x) = \frac{2x+3}{x+2}$, find $f^{-1}(x)$.
- a. $\frac{x+2}{2x+3}$ b. $\frac{2x-3}{x-2}$ c. $\frac{3-2x}{2+x}$
d. $\frac{3-2x}{x-2}$ e. $\frac{-2x-3}{-x-2}$
24. Solve for x : $\log_6(x-3) + \log_6(x-4) = 1$.
- a. $x = 1, 6$ b. $x = 3, 4$ c. $x = -3, 4$
d. $x = 4$ e. $x = 6$

25. Solve for θ : $4\sin^2 \theta = 1 + 4\cos \theta$, where $0^\circ \leq \theta \leq 360^\circ$.
- a. $\theta = 60^\circ, 300^\circ$ b. $\theta = 30^\circ, 150^\circ$ c. $\theta = 30^\circ, 330^\circ$
d. $\theta = 60^\circ, 120^\circ$ e. $\theta = 120^\circ, 240^\circ$
26. In $\triangle ABC$, if $\overline{AC} = 50$ feet, $\overline{BC} = 70$ feet, and $m\angle ACB = 70^\circ$, find \overline{AB} .
- a. 70.75 feet b. 86.02 feet c. 48.99 feet
d. 78.76 feet e. 98.97 feet
27. The range of $f(x) = \frac{2x+1}{x-1}$ is the set of numbers q such that
- a. $q \neq 2$ b. $q \neq 1$ c. $q \neq \frac{-1}{2}$
d. $q \neq \frac{-1}{2}, 1$ e. q is any real number
28. In $\triangle ABC$, let D be between B and C , and let E be between A and C . If $m\angle B = m\angle C$, $\overline{AD} = \overline{AE}$, and $m\angle BAD = 40^\circ$, find $m\angle CDE$.
- a. 10° b. 20° c. 30°
d. 40° e. 60°
29. Find the distance from the point $(4,3)$ to the line $y = -x + 3$
- a. 0 b. $2\sqrt{2}$ c. 2
d. $\sqrt{10}$ e. 4

30. Ike plays the following game. He will flip a fair coin until the second time that it lands heads up. What is the probability that he flips the coin exactly three times?

- a. 1 b. $\frac{1}{2}$ c. $\frac{1}{3}$
 d. $\frac{1}{4}$ e. $\frac{3}{8}$

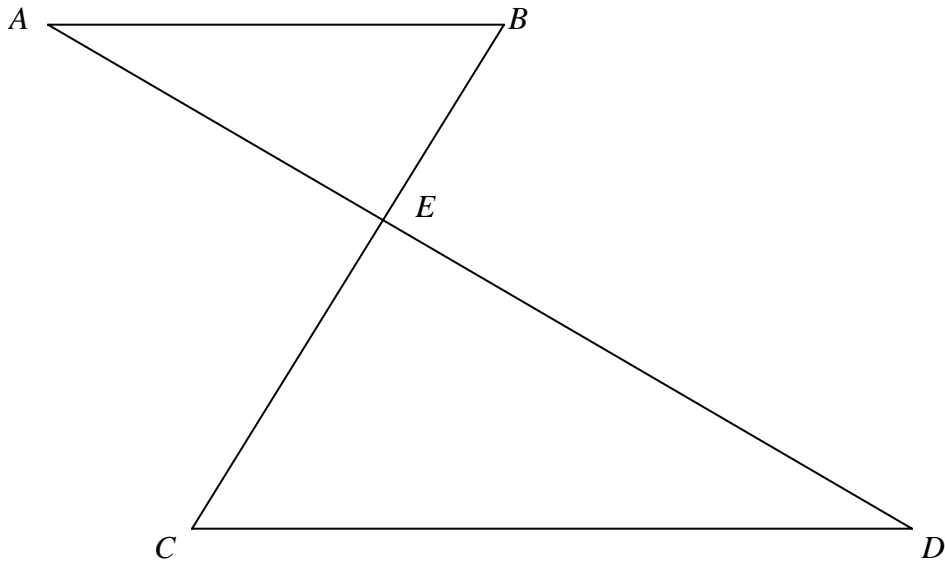
31. Twelve rabbits are introduced onto an island that previously had no rabbits. Three years later there are 30 rabbits on the island. Assuming exponential growth, how many rabbits are on the island twelve years after their introduction? Round off your answer to the nearest rabbit.

- a. 39 b. 120 c. 188
 d. 469 e. 1172

32. For the figure below,

- i) $\overline{AB} \parallel \overline{CD}$ ii) $\overline{AB} = 5$ iii) $\overline{CD} = 9$ iv) $\overline{AD} = 28$.

Find \overline{AE} .



- a. 10 b. 14 c. 18
 d. $\frac{140}{9}$ e. None of the above