

# Algebra Second Course/ Integrated Math III

**2006**

Sponsored by the Indiana Council of Teachers of Mathematics

Indiana State Mathematics Contest

This test was prepared by faculty at **Indiana University Northwest**

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Next year's math contest date: April 21, 2007

- 1) A 50 year old man has two children. One of the children is 4 years older than the other. Eight years from now, the father will have the same age as the sum of the ages of the children. What is the age of the younger child now?

A) 14                      B) 16                      C) 19                      D) 24                      E) 30

- 2) When  $x^5 - 6x^3 + 5x$  is factored completely with integer coefficients, then the sum of the factors is

A)  $x^2 + 3x - 5$                       B)  $x^2 + 2x - 5$                       C)  $x^2 + x - 7$

D)  $x^3 + x + 5$                       E) none of these

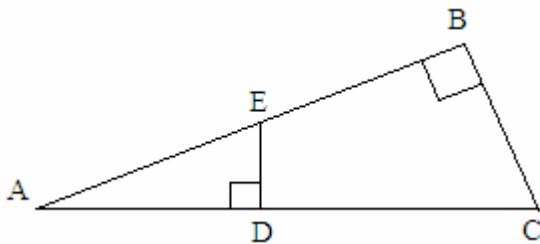
- 3) The ratio of boys to girls in a certain class is  $\frac{2}{3}$ . If 4 more boys and 4 more girls are added to the class, the ratio of boys to girls will be  $\frac{5}{7}$ . How many boys are in the original class?

A) 16                      B) 20                      C) 24                      D) 28                      E) 32

- 4) The equation  $\sqrt{2x-1} - \sqrt{x-1} = 1$  has two solutions. The sum of these solutions is

A) 3                      B) 4                      C) 5                      D) 6                      E) 7

- 5) In the figure below, triangles ABC and ADE are right triangles with length of  $AB = 24$ , length of  $BC = 10$ , length of  $AC = 26$ , and length of  $AD = x$ . Find  $x$  to the nearest tenth so that the area of triangle ADE is 6 square units.



A) 1.2                      B) 2.2                      C) 2.4                      D) 5.4                      E) 6.0

- 6) Let M and N denote the midpoints of two consecutive sides of a rectangle. The segment MN divides the area of the rectangle in the ratio

A) 7 : 1.                      B) 4 : 1.                      C) 7 : 4.                      D) 4 : 3.                      E) none of these

- 7) Simplify the following rational expression:  $\frac{a^3 - b^3}{a^4 + a^2b^2 + b^4}$
- A)  $\frac{a - b}{a^2 + ab + b^2}$       B)  $\frac{a - b}{a^2 - ab + b^2}$       C)  $\frac{1}{a^2 + ab + b^2}$
- D)  $\frac{1}{a^2 + b^2}$       E)  $\frac{a + b}{a^2 - ab + b^2}$
- 8) A random sample of frogs is taken in a section of rain forest in Brazil. The sample contains 1 red frog, 6 blue frogs, and 45 green frogs. Researchers believe that an acre of this rainforest can sustain 25,000 frogs. Assuming that researchers are correct, use the ratios obtained from the sample to find how many blue frogs are in an acre?
- A) 481      B) 1542      C) 2885      D) 4167      E) 7168
- 9) A small square lies inside a larger square whose perimeter is four times bigger than the perimeter of the small one. What is the area of the smaller square as a percentage of the area of the larger one?
- A) 3.125%      B) 6.25%      C) 12.5%      D) 25%      E) none of these
- 10) Given  $x^2 + y^2 = 10$  and  $\sqrt[4]{xy} + \sqrt{xy} + 27 = 29$ , what is one value of  $x + y$ ?
- A)  $2\sqrt{3}$       B)  $2\sqrt{2}$       C) 0.5      D)  $\frac{\sqrt{3}}{3}$       E) 8
- 11) Solve *completely* for  $x$ :  $3^{x^2} = 9^{x+4}$
- A) -4      B) -2      C) -4, 2      D) -2, 4      E) none of these
- 12) For the function  $f(x) = x^4 + ax^3 + bx^2 + cx + 2$ , where -1 is a triple root of  $f(x)$ , what is  $a$ ?
- A) -2      B) -3      C) 2      D) 5      E) 6

- 13) Whitney has a rectangular garden that is 16 feet long and 8 feet wide. She wants to put a strip of gravel of uniform width around the garden. She has enough gravel to cover 112 square feet. How wide should she make the strip?
- A) 1 foot      B) 2 feet      C) 3 feet      D) 4 feet      E) 5 feet
- 14) Solve *completely* for  $x$ :  $\log_2 x + \log_2(x-7) = 3$
- A) -1, 4      B) 1, 8      C) 8      D) 16      E) none of these
- 15) Find a pair of numbers A and B so that after performing the addition  $\frac{A}{x^2-1} + \frac{B}{x^2+3x+2}$ , the result reduces to  $\frac{6}{x^2+x-2}$ .
- A) A = 1 and B = 5      B) A = 2 and B = 4      C) A = 3 and B = 3  
D) A = 4 and B = 2      E) A = 5 and B = 1
- 16) Find the length of a secant of the larger of the two concentric circles with radii 10cm and 6 cm, if the secant is tangent to the smaller circle.
- A) 8 cm      B) 12 cm      C) 16 cm      D)  $12\pi$  cm      E)  $16\pi$  cm
- 17) Jane filled her beaker with 40 ml of a 75% solution of hydrochloric acid. Her chemistry teacher pointed out that she should have 40 ml of a 50% hydrochloric acid solution. Jane drains  $x$  ml from her beaker and replaces them with  $x$  ml of water to get the proper solution. In which of the following ranges does  $x$  lie?
- A) Less than 5      B) At least 5, but less than 10  
C) At least 10, but less than 15      D) At least 15, but less than 20  
E) 20 or more
- 18) How many times will the graph of  $|x| + |y| = 2$  intersect the graph of  $x^2 - y = 2$ ?
- A) 0      B) 2      C) 3      D) 4      E) 5

19) The solution of the inequality  $\sqrt[3]{2+x+x^2} \leq 2$  is

- A) 0                      B)  $-3, 2$                       C)  $x \geq -3$                       D)  $2 \leq x$                       E)  $-3 \leq x \leq 2$

20) The graph of function  $y = ax$ , where  $a > 0$ , bisects an angle made by the graphs of  $y = \frac{1}{3}x$  and  $y = -3x$ . Find the value of  $a$ .

- A) 0                      B) 1                      C) 2                      D) 3                      E) none of these

21) Suppose the vertex of the curve of  $y = (a+x)^2 + 2(a+1)x + 6$  is  $(3, 1)$ . Find the value of  $a$ .

- A) 0                      B)  $-1$                       C)  $-2$                       D)  $-3$                       E)  $-4$

22) Suppose  $f\left(\frac{x+1}{2}\right) = 3x+2$ . Find  $f\left(\frac{1-3x}{2}\right)$ .

- A)  $2-9x$                       B)  $-1-4x$                       C)  $6x-3$                       D)  $6x+3$                       E) none of these

23) In an arithmetic sequence, the first term is  $\frac{1}{10}$  and the hundredth term is  $\frac{2}{10}$ . Find the sum of the first 50 terms.

- A)  $5 + \frac{245}{198}$                       B)  $5 + \frac{392}{330}$                       C)  $1 + \frac{49}{990}$   
D)  $\frac{392}{330}$                       E) none of these

24) How many real solutions does the equation  $x^4 + 6x = 1$  have?

- A) 0                      B) 1                      C) 2                      D) 3                      E) 4

25) The domain of the function  $f(x) = \frac{x}{\sqrt{1-x}} + \frac{2x}{\sqrt{2-x}}$  is

- A)  $x < 1$                       B)  $x < 2$                       C)  $1 < x < 2$   
D)  $x < 1$  and  $x \neq 0$                       E)  $x < 2$  and  $x \neq 0$

26) Find the product of the solutions to the equation  $2x^3 + x^2 - 13x + 6 = 0$ .

- A)  $-6$       B)  $-3$       C)  $3$       D)  $6$       E)  $12$

27)  $4^{\log_2\left(\frac{1}{4}\right)} =$

- A)  $-8$       B)  $\frac{1}{16}$       C)  $\frac{1}{4}$       D)  $2$       E)  $4$

28) The product of the solutions of the equation  $\ln(x^2) = (\ln x)^2$  is

- A)  $2$       B)  $e$       C)  $\pi$   
D)  $e^2$       E) The equation has no solutions.

29) Jim drove to his friend's house at 30 mph. How fast does he need to drive home so that he can average 40 mph for the round trip?

- A) 40 mph      B) 45 mph      C) 50 mph      D) 55mph      E) 60 mph

30) Suppose  $f(x) = 2x + 1$  and  $g(x) = x - 3$ . Find the function  $h(x)$  such that  $f \circ h = g$ .

- A)  $2x - \frac{1}{2}$       B)  $2x - 1$       C)  $\frac{1}{2}x - 2$       D)  $\frac{1}{2}x + 1$       E) none of these

31) Let  $a$ ,  $b$  and  $c$  be elements of the set  $\{-1, 1\}$ , such that  $ab^2c^3 = 1$ . Find  $|ab + ac + bc - 1|$

- A)  $-1$       B)  $0$       C)  $1$       D)  $2$       E) Cannot be determined.

32) Let  $(x, y)$  be a point in the solution set of following system of inequalities:

$$\begin{cases} x^2 + y^2 < 1 \\ 1 < |x| + |y| \end{cases}$$

What is the probability that both  $x$  and  $y$  are negative?

- A)  $0$       B)  $\frac{1}{4}$       C)  $\frac{1}{\pi}$       D)  $\frac{3}{4}$       E)  $1$

- 33) Consider the equation  $m^2(n+1) = 80$ , where  $m$  and  $n$  are positive integers. How many solutions  $(m, n)$  does it have?
- A) 1                      B) 2                      C) 3                      D) 4                      E) Infinitely many.
- 34) Two friends were traveling together. One had 2 loaves of bread and the other had 3. They met a hungry person. They all set together and ate the same amount of bread until they finished all the loaves. The hungry person gave them a total of \$5 for his share. What is the fair way of dividing this amount of money between the two friends?
- A) \$1.00 and \$4.00                      B) \$2.50 and \$2.50                      C) \$2.00 and \$3.00
- D) \$1.50 and \$3.50                      E) none of these
- 35) Given that  $7200 = 2^5 \cdot 3^2 \cdot 5^2$ , how many positive integers will divide evenly into 7200?
- A) 9                      B) 20                      C) 30                      D) 49                      E) 54
- 36) Suppose the function  $f$  satisfies the following conditions  $f(1) = 4$ ,  $f(x+y) = f(x) + f(y)$ .  
Find  $f(519) + f\left(\frac{519}{2}\right) + f\left(\frac{519}{4}\right)$ .
- A) 3633    B) 3533                      C) 3333                      D) 3033                      E) none of these