

Algebra I 2004

Sponsored by the Indiana Council of Teachers of Mathematics
Indiana State Mathematics Contest
This test was prepared by Indiana University Northwest

Indiana State Mathematics Contest

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Next year's math contest date: April 23, 2005

Indiana Council of Teachers of Mathematics
Algebra I
Indiana State Math Contest 2004
Indiana University Northwest

1. Find the slope of the line $2x - 5y = 10$
(A) $-\frac{2}{5}$ (B) $\frac{5}{2}$ (C) $\frac{2}{5}$
(D) -2 (E) 5

2. Find all real x so that the following statement is true: $\frac{x}{2} - 1 \leq 1 - \frac{x}{2}$.
(A) all real numbers (B) no real number (C) $x \leq -2$
(D) $x \leq 0$ (E) $x \leq 2$

3. The convenience store inventory showed that they had 120 candy bars when an order for six crates arrived. There were a dozen boxes in every crate and there were two dozen candy bars in each box. How many candy bars, to the nearest dozen, did the convenience store have after the order came?
(A) 1848 (B) 264 (C) 154
(D) 144 (E) 22

4. The result of calculating the value of $\frac{(2 \times 10^{52})(3 \times 10^{48})(4 \times 10^{-35})}{13 \times 10^{76}}$ is
(A) less than $-100,000$ (B) between $-100,000$ and 0 (C) between 0 and 1
(D) between 1 and $100,000$ (E) greater than $100,000$

5. Solve for x : $2(x - 5)(x + 3) = -28$
(A) $\{-2, 5\}$ (B) $\{-3, -14\}$ (C) $\{\frac{4 \pm \sqrt{3}}{2}\}$
(D) $\{2 \pm \sqrt{2}\}$ (E) $\{1 \pm \sqrt{2}\}$

6. Mary's summer outfit consists of one blouse, one skirt, one scarf, and a pair of shoes. She has 2 purple blouses, 3 blue blouses, 3 black skirts, 1 white skirt, 5 scarves, 2 black pairs of shoes, and a pair of white shoes. How many different outfits can she wear if black skirts can be worn only with black shoes and white skirt can be worn only with white shoes?
(A) 175 (B) 150 (C) 125 (D) 100 (E) 75

7. Which of the following is a factor of: $\frac{a^2}{16} - \frac{4b^2}{9}$?

(A) $\frac{a}{4} + \frac{2b}{9}$

(B) $\frac{a}{4} - \frac{2b}{9}$

(C) $\frac{a}{4} - \frac{2b}{3}$

(D) $\frac{a}{2} - \frac{2b}{3}$

(E) $\frac{a}{2} + \frac{2b}{3}$

8. Line L passes through the origin and is parallel to the line $y = \frac{2}{3}x - 6$. What is the sum of the

coordinates of the point where the line L intersects the line $y = \frac{1}{2}x - 4$?

(A) -40

(B) -10

(C) 0

(D) 12

(E) 20

9. Solve for y in terms of x if $x = \frac{2}{1-2y}$

(A) $\frac{2+x}{2x}$

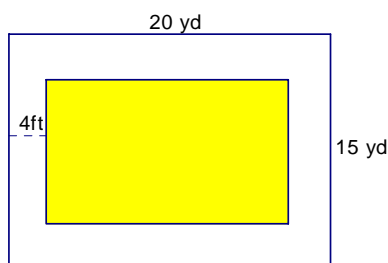
(B) $\frac{2x-1}{2x}$

(C) $\frac{2-x}{2x}$

(D) $\frac{x-2}{x}$

(E) $\frac{x-2}{2x}$

10. A rectangular swimming pool is surrounded by a wooden walkway 4 feet wide and then enclosed by a chain link fence next to the walkway. If the length of the fence is 20 yards and its width is 15 yards, what is the area of the wooden walkway to the nearest square foot?



(A) 776

(B) 800

(C) 1924

(D) 2296

(E) 2700

11. Solve the following system for x : $\begin{cases} 2x - 3y = 5 \\ -\frac{x}{2} + \frac{5y}{2} = \frac{1}{2} \end{cases}$

(A) 1

(B) 2

(C) 3

(D) 4

(E) 5

12. Find the equation of the line that passes through the point $(-2,3)$ that is parallel to the graph of $2x - 3y = 6$.

(A) $y = -\frac{3}{2}x - 3$

(B) $y = -\frac{2}{3}x + 1$

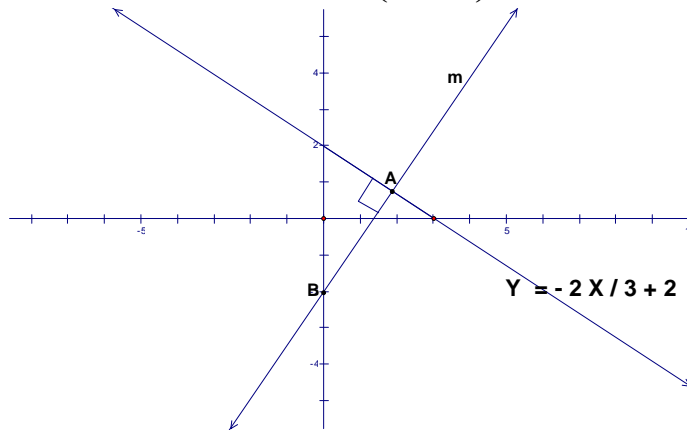
(C) $y = \frac{2}{3}x + \frac{13}{3}$

(D) $y = -\frac{2}{3}x + 3$

(E) $y = \frac{3}{2}x + 6$

13. Find the equation of the line m in the following picture that is perpendicular to the line $y = -\frac{2x}{3} + 2$

and also passes through the points $A\left(\frac{24}{13}, \frac{10}{13}\right)$ and $B(0, -2)$



(A) $y = \frac{3x}{2} - 2$

(B) $y = -\frac{2x}{3} - 2$

(C) $y = \frac{3x}{2} + 2$

(D) $y = \frac{2x}{3} + \frac{82}{39}$

(E) $y = \frac{2x}{3} - \frac{18}{39}$

14. If r and s represent any real numbers, which of the following statements are always true?

i) $(rs^2)^2 = r^2s^4$

ii) $(r^3s^2)^3 = r^{18}$

iii) $\sqrt{a^{16}} = a^4$

(A) i only

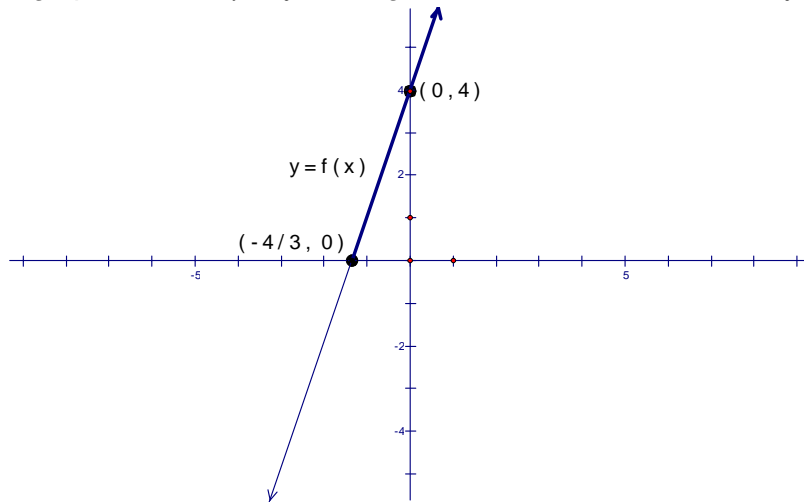
(B) ii only

(C) iii only

(D) i and iii

(E) i, ii, and iii

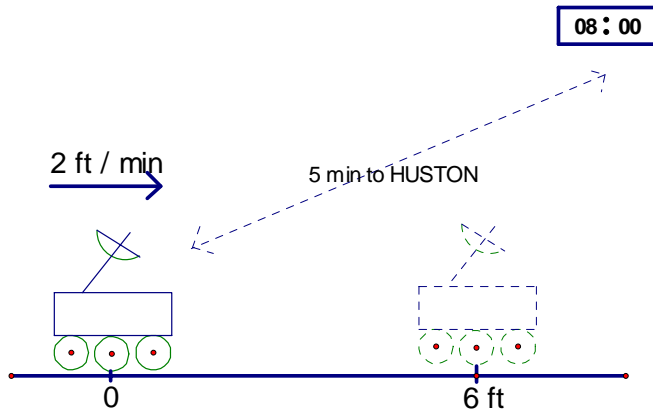
15. The graph of a line $y = f(x)$ is given below. Find all x so that $f(x) \geq 0$



- (A) $y \geq 4$ (B) $x < -\frac{4}{3}$ (C) $x \geq -\frac{4}{3}$
 (D) $(-\infty, -\frac{4}{3}]$ (E) $[4, +\infty)$

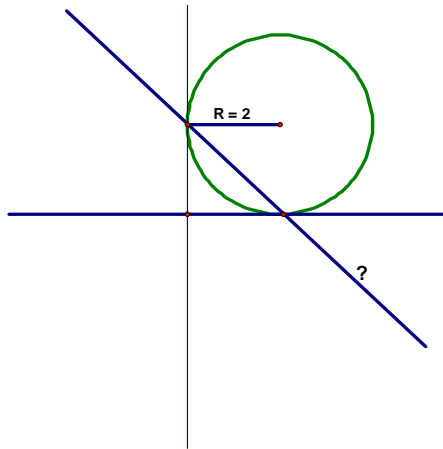
16. Suppose that a Mars Rover travels at $2 \frac{\text{ft}}{\text{min}}$ and that it takes 5 minutes for a signal from Huston to reach the Rover. Huston sends the signal at 8:00 GMT for the Rover to go forward. The Rover needs to move 6 feet and stop.

At what time should Huston send the stop signal?



- (A) 08:03 GMT (B) 08:06 GMT (C) 08:07 GMT (D) 08:08 GMT (E) 08:10 GMT

17. The circle in the picture intersects the coordinate axes at $(a, 0)$ and $(0, b)$. Write the equation of the line going through the intercepts.



- (A) $y = -x - 1$ (B) $y = -x + 2$ (C) $y = x - 2$
 (D) $y = -ax + b$ (E) $y = ax + b$

- 18) If $a @ b = ab + 1$ and $a \# b = a + b$ find the value of $4 @ [(6 \# 8) \# (3 @ 5)]$.

- A) 34 B) 76 C) 116 D) 120 E) 121

- 19) Find the value of $z + w$ if $z = \sqrt{x+33}$ and $w = \sqrt{x-12}$, where w , x and z are integers.

- A) 16 B) 9 C) 35 D) 15 E) 7

- 20) If $2x = 2y + 16$ and $3y = -24 + 3x$, then $x - y$ has what value?

- A) There is no solution. B) There are infinitely many solutions.
 C) 8 D) 16 E) -8

- 21) Find y , if the distance between $(3, y)$ and $(6, -4)$ is $\sqrt{34}$.

- A) 6 or -5 B) 5 or -6 C) 3 or -4 D) -9 or 1 E) 9 or -1

- 22) The population of Burma in 1981 was about 35 million. Which of the following is the correct value for the approximate population in scientific notation?

- A) 3.5×10^3 B) 3.5×10^7 C) 3.5×10^6 D) 3.5×10^5 E) 35×10^6

23) Magic squares are square arrays of numbers. The sum of numbers in each row, diagonal and column are the same. Determine the value

1	2	a
-4	0	4
3	b	-1

of the variables given, listed in order (a,b).

- A) (2, -3) B) (3,2) C) (-3, 2) D) (-3, -2) E) (-2, -3)

24) Factor $a^2 - b^2$

- A) $(a-b)^2$ B) $(a-b)(a+b)$ C) $a^2 - 2ab + b^2$ D) $(a - 2ab + b)^2$ E) $(a - ab + b)^2$

25. Simplify $\sqrt[4]{16a^{64}}$

- (A) $2a^8$ (B) $2a^{16}$ (C) $4a^8$ (D) $4a^{16}$ (E) $64a^{256}$

26. If a line through $(-6, 10)$ and $(x, -10)$ has a slope of 4, then the value of x is:

- (A) $\frac{10}{3}$ (B) -11 (C) -12 (D) -15 (E) -20

27) Henry can repair a car alone in 7 hours. Cheryl can repair the same car alone in 5 hours. How long would it take them to repair the car together?

- A) $2\frac{11}{12}$ hours B) 6 hours C) $3\frac{4}{7}$ hours D) $3\frac{1}{12}$ hours E) $2\frac{5}{6}$ hours

28) Which of the following numbers are irrational?

- A) .424242... B) .121231234... C) $\overline{.12}$ D) -.000033 E) -.53698234

29) Determine the answer: $\sqrt{3ab} \cdot \sqrt{3a^2} \cdot \sqrt{3a^4b}$

- A) $3a^3\sqrt{3ab}$ B) $3a^3b\sqrt{3ab}$ C) $9a^3b\sqrt{3a}$ D) $3a^3b\sqrt{3a}$ E) $3a^3b^2\sqrt{3a}$

30) Given $z = x^2 + 3x + \frac{y^2 + 2xy}{x+2}$, find the value of z when $x = 2$.

- A) $\frac{y^2 + 4y + 10}{4}$ B) $\frac{y^2 + 4y + 32}{2}$ C) $\frac{y^2 + 4y + 40}{4}$ D) $y^2 + y + 10$
E) $y^2 + y + 40$

31) A one-foot stick is broken twice to form three pieces each an integral number of inches long. The three pieces are then arranged to form a triangle. How many different triangles could be formed in this fashion?

- A) 1 B) 2 C) 5 D) 4 E) 3

32) Two students started walking from the same point in opposite directions for 12 feet each. Each then made a 90 degree turn to the left and walked another 5 feet. How far apart are the two students?

- A. 13 ft. B. 24 feet C. 34 feet D. 26 feet e. None of these

33. 15 people can paint 75 houses in 150 days. In how many days can 5 people paint 25 houses?

- A. 50 days B. 450 days C. 150 days D. 100 days E. None of these

34. Sammy likes the number 400 but does not like the number 300; she likes the number 100 but does not like the number 99; she likes the number 2500 but does not like the 2100. Which of the following numbers does Sammy like?

- A. 900 B. 1000 C. 1100 D. 1200 E. None of these

35. Simplify the expression $\frac{a}{b} - \frac{b}{a} =$

- (A) 0 (B) -1 (C) $a - b$ (D) $\frac{a^2 - b^2}{ab}$ (E) $\frac{b^2 - a^2}{ba}$

36. Juan wanted to purchase 2 dozen pencils and a pen. Those items cost \$8.45 and he did not have enough money. So he decided to purchase 8 fewer pencils and paid \$6.05. How much did the pen cost?

- A. \$1.25 B. \$.30 C. \$1.65 D. \$1.15 E. \$0.90

37. Hose A takes 45 minutes to fill the bucket with water. Hose B can do the same in 35 minutes. If you use both hoses, how long will it take to fill the bucket?

- A. $19\frac{7}{16}$ minutes B. $19\frac{11}{16}$ minutes C. $18\frac{13}{16}$ minutes D. $20\frac{1}{16}$ minutes E. $20\frac{9}{16}$ minutes

38. You were supposed to add A and B. But by accident, you subtracted B from A and got 4. This number is different from the correct answer by 12. What is A?

- A. $A=4$ B. $a=5$ C. $A=6$ D. $A=10$ E. $A=16$

39. It is 7 miles between Joe's house and Larry's house. Joe and Larry started from their own houses, walking to each other's houses, at noon, and then meeting at 1:30 pm. Joe walked 1 mile per hour faster than Larry. How fast did Larry walk?

- A. $3\frac{1}{3}$ mph B. $2\frac{2}{3}$ mph C. $1\frac{5}{6}$ mph D. $2\frac{3}{7}$ mph E. $2\frac{6}{7}$ mph

40. A classroom contained an equal number of boys and girls. Eight girls left to play hockey, leaving twice as many boys as girls in the classroom. What was the original number of students present?

- A. 16 students B. 12 students C. 32 students D. 24 students E. None of these

41. The median of 4 numbers is 10. If three of the four numbers are 3, 12 and 21, what is the fourth number?

- A. 4 B. 6 C. 8 D. 9 E. 7

42) Tamika told her friend Jared, "I have a handful of coins, but I cannot make change for a dollar bill." She had no half-dollars or dollar coins. What is the maximum amount of money that Tamika could have had in her hand?

- A) \$.99 B) \$1.14 C) \$1.09 D) \$1.19 E) \$1.24

43) A bag contains one blue, three orange, and three brown marble. Find the probability that Omar picks a blue marble on his first draw from the bag.

- A) 0.143 B) 0.167 C) $\frac{1}{4}$ D) $\frac{1}{6}$ E) $\frac{1}{7}$

44) Ms. Jackson teaches a class of 32 students. There are 17 boys in the class and their average score on the first test is 85%. If the average of the entire class is 89% for this test, what is the average score for the girls in the class? (Round to the nearest whole percent.)

- A) 94% B) 93% C) 95% D) 92% E) 96%