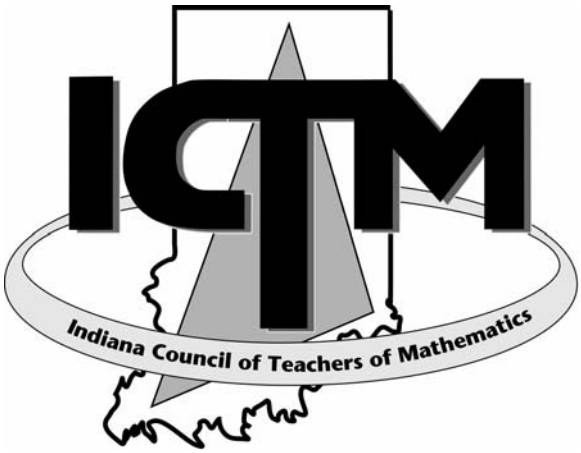


ALGEBRA 2 CONTEST

2004

Sponsored by the Indiana Council of Teachers of Mathematics

Indiana State Mathematics Contest

This test was created by Indiana University Northwest Mathematics Department Faculty

ICTM Website

http://www.indianamath.org/

Do not open this test booklet until you have been advised by the test proctor.

Next year's math contest date: April 23, 2005

INDIANA STATE MATHEMATICS CONTEST
ALGEBRA 2 EXAM
2004
(Created by IUN Mathematics Department Faculty)

1) If $|1-2x| \le 5$, then

(A) $-2 \le x \le 3$ (B) $-3 \le x \le 2$ (C) $-3 \le x \le 3$

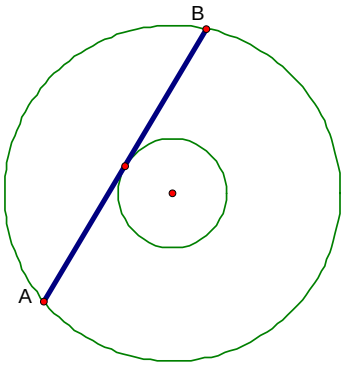
(D) $x \le -2$ (E) $x \ge 2$

2) If x and y vary indirectly and $x = -2$ when $y = -8$, then what is x when $y = 10$?

(A) -40 (B) $\frac{1}{4}$ (C) $\frac{2}{5}$

(D) $\frac{8}{5}$ (E) 40

3) Find the length of the segment AB if it is tangent to the small circle of radius 2 cm and is a chord of the large circle of radius 6 cm. (The two circles are concentric.)



(A) 12 cm (B) $8\sqrt{3}$ cm (C) 14 cm

(D) $8\sqrt{2}$ cm (E) can not be determined

4) Find all x so that $\frac{x-2}{x+4} < 0$.

(A) $x < -4$ (B) $x < -2$ (C) $x > 4$

(D) $-2 < x < 4$ (E) $-4 < x < 2$

5) Suppose $f(x) = 2x - 5$ and $f(g(x)) = x = g(f(x))$. Then $g(x) =$

(A) $\frac{5-x}{2}$ (B) $\frac{x+2}{-5}$ (C) $\frac{x+5}{2}$

(D) $\frac{1}{2}x + 5$ (E) $\frac{1}{5}x - 2$

6) Solve for *x*: $2(x-5)(x+3) = -28$

(A) 5, – 3 (B) 5, – 3, – 14 (C) $1 \pm \sqrt{2}$

(D) $2 \pm \sqrt{2}$ (E) $\frac{4 \pm \sqrt{3}}{2}$

7) How many different four-digit numbers can be formed by using the digits 1, 2, 3, 4 if no digit may be used twice and 2 must be next to 4? (Example: 1243)

(A) 10 (B) 12 (C) 16

(D) 20 (E) 40

8) Which of the following is a factor of $8p^3 + q^3$?

(A) $2p - q$ (B) $4p^2 + q^2$ (C) $4p^2 - 2pq + q^2$

(D) $p^2 + 2pq + q^2$ (E) $4p^2 - q^2$

9) Solve the equation, $9^{x^2} = 3^{2x}$. The sum of its solutions is

(A) –1 (B) 0 (C) 1

(D) 2 (E) 3

10) Solve for *t* in terms of *u* if $u = \frac{3t-4}{t+2}$.

(A) $\frac{2u+4}{3-u}$ (B) $\frac{4-2u}{u+3}$ (C) $\frac{2-3u}{4u+1}$

(D) $\frac{3u+4}{u-2}$ (E) $\frac{1-4u}{3u-2}$

11) A rectangular frame surrounds a 16 inch by 20 inch picture. The area of the frame (not including the picture) is160 in^2. The width of the frame (measured from the edges of the picture to the outside of the frame) is x. What is x to the nearest tenth of an inch?

(A) 1.6 (B) 2.0 (C) 2.4 (D) 2.5 (E) 3.0

12) If $\log_2 x = a$, then $\log_2(4x^3) =$

(A) $12a$ (B) $2 + 3a$ (C) $6 + a$ (D) $2 + a^3$ (E) $2a^3$

13) If $\left\{\begin{matrix} 2a - b = 3 \\ 4a + 3b = 7 \end{matrix}\right\}$, then $b =$

(A) $-\frac{1}{2}$ (B) 0 (C) $\frac{1}{2}$

(D) $\frac{1}{2}$ (E) $\frac{1}{5}$

14) The sum of the coordinates of the point of intersection of the lines $\frac{1}{2}x - \frac{1}{3}y = 1$ and $\frac{2}{5}x - \frac{1}{2}y = \frac{1}{2}$ is

(A) $-\frac{20}{7}$ (B) $-\frac{9}{7}$ (C) $\frac{9}{7}$

(D) $\frac{20}{7}$ (E) $\frac{29}{7}$

15) If m and n are positive integers such that $2m + 3n = 12$, then what is the value of $m+n$?

(A) 6 (B) 5 (C) 4

(D) 2 (E) cannot be determined

16) The arithmetic mean of the *x*-coordinates of the points where the graph of $f(x) = |x-2| - 1$ intersects the *x*-axis is

(A) 2 (B) $\frac{5}{4}$ (C) 1

(D) 0 (E) none of these

17) Jill ran the first one half of the race at 12 mph; she ran the third fourth at 10 mph; she ran the last fourth of the race at 12 mph. What was her average speed for the race?

(A) 12.2 mph (B) 11.8 mph (C) 11.6 mph

(D) 11.4 mph (E) 11.2 mph

18) How much water must we add to 40 liters of a 10% salt solution to obtain a 5% salt solution?

(A) 10 liters (B) 20 liters (C) 40 liters

(D) 50 liters (E) 60 liters

19) A square is inscribed in a circle of radius *r*. What is the area of the square?

(A) $2\pi r^2$ (B) πr^2 (C) $3r^2$

(D) $2r^2$ (E) r^2

20) The ratio of men to women in the US Senate is 43 to 7. If there are 100 senators, how many are women?

(A) 7 (B) 13 (C) 14

(D) 43 (E) 86

21) Suppose $a^* = 2 - a$. Find $\left((-1)^*\right)^*$.

(A) -3 (B) -2 (C) -1

(D) 0 (E) 1

22) Let $\log\left(x^2 - 1\right) - \log\left(x - 1\right) = 2$. Find x.

(A) 80 (B) 99 (C) $2^{10} + 1$

(D) $2^{10} - 1$ (E) 101

23) A teacher knows that 15 among her 35 Algebra II students have seen the movie, *The Lord of the Rings: The return of the King*, and 12 have seen *Finding Nemo*. If 14 students saw neither of these two movies, how many of her students saw both?

(A) 2 (B) 4 (C) 6

(D) 8 (E) cannot be determined

24) In how many ways can you write 53 as a sum of two primes?

(A) 0 (B) 1 (C) 2

(D) 3 (E) more than 3

25) If $x + y = i$ and $xy = i$, where $i = \sqrt{-1}$, determine the value of $x^3 + y^3$.

(A) $3 + i$ (B) $1 + i$ (C) 6 (D) 2 (E) $3 - i$

26) Mrs. Math leaves her house for school every morning at the same time. When she averages 30 miles per hour, she arrives at her school 6 minutes late. When she averages 48 miles per hour, she arrives 7.5 minutes early. At what average speed, in miles per hour, should Mrs. Math drive to arrive at her school precisely on time?

(A)32 (B)36 (C)40 (D)44 (E)48

27) The equation $ax^2 + 4 = \sqrt{2}\,x$ has zero discriminant, where a is a real number. Find the root(s) of the equation.

(A) $2\sqrt{2}$ (B) $2\sqrt{2}$ and $-2\sqrt{2}$ (C) 2 and -4

(D) $4\sqrt{2}$ (E)4

28) Five test scores have a mean of 83, a median of 85 and a mode (most frequent score) of 95. The sum of the two lowest test scores is

(A)99 (B)112 (C)135 (D)140 (E)142

29) A non-constant polynomial function $f(x)$ satisfies
$f(-3) = f(-1) = f(2) = f(4) = 5$
What is the smallest possible degree of $f(x)$?

(A)1 (B)2 (C)3 (D)4 (E)5

30) If $2004^{10x-1} = 1$, what is $\log x$?

(A) 3.3019 (B) 1 (C) -1 (D) –3.3019 (E) -4

31) If x is 120% of y, what percent of $4x$ is $3y$?

(A) 62.5% (B)75% (C)80% (D)83.33% (E)133.33%

32) Find a value of c if $(c + ci)^{2004} = 1$ and $i = \sqrt{-1}$.

(A) $\frac{1-i}{2}$ (B) $\frac{2}{1-i}$ (C) $\frac{1}{2} - i$ (D) $2 + i$ (E) $\frac{1}{2} + i$

33) What is the sum of the real solutions of equation,

$$\mathbf{2}\,x^4 + \mathbf{0}\,x^3 + \mathbf{0}\,x^2 + \mathbf{4}\,x = 0$$

(A) -1.2599 (B) -2.004 (C) 2.0004 (D) -3.3408 (E) 4.002

34) Determine the equation of the ellipse having vertices $(\pm 10, 0)$ and foci $(\pm 6, 0)$.

(A) $\frac{x^2}{100} + \frac{y^2}{36} = 1$ (B) $\frac{x^2}{100} - \frac{y^2}{36} = 1$ (C) $100x^2 + 36y^2 = 1$

(D) $\frac{x^2}{100} + \frac{y^2}{64} = 1$ (E) $\frac{x^2}{36} + \frac{y^2}{64} = 1$

35) The radius of a bicycle wheel is $\frac{7}{2\pi}$ ft. The number of revolutions it will make in going 91ft is

(A) 10 (B)11 (C)12 (D)13 (E)14

36) The third term of an arithmetic sequence is 11 and the sixth term is 23. Find the twenty-ninth term of the sequence.

(A) 63 (B)115 (C)141 (D)163 (E)231